Fourth-Year Courses
Proposed Standards

Revised April 5, 2018

Bridge Math | B

Bridge Math is a course intended to build upon concepts taught in previous courses to allow students to gain a deeper knowledge of the real and complex number systems as well as the structure, use, and application of equations, expressions, and functions. Functions emphasized include linear, quadratic and polynomial. Students continue mastery of geometric concepts such as similarity, congruence, right triangles, and circles. Students use categorical and quantitative data to model real life situations and rules of probability to compute probabilities of compound events.

Bridge Math includes the following domains and clusters:

- **The Real Number System**
  - Use properties of rational and irrational numbers.
- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **The Complex Number System**
  - Perform arithmetic operations with complex numbers.
- **Seeing Structure in Expressions**
  - Write expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Perform arithmetic operations on polynomials.
  - Understand the relationship between zeros and factors of polynomials.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Solve equations and inequalities in one variable.
  - Solve systems of equations.
  - Represent and solve equations and inequalities graphically.
- **Interpreting Functions**
  - Understand the concept of a function and use function notation.
  - Interpret functions that arise in applications in terms of the context.
  - Analyze functions using different representations
- **Similarity, Right Triangles, and Trigonometry**
  - Understand similarity in terms of similarity transformations.
  - Define trigonometric ratios and solve problems involving right triangles.
- **Circles**
  - Find arc lengths and areas of sectors of circles.
- **Geometric Measurement and Dimension**
  - Visualize relationships between two-dimensional and three-dimensional objects.
- **Modeling with Geometry**
  - Apply geometric concepts in modeling situations.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.
  - Interpret linear models.
- **Conditional probability and the Rules of Probability**
  - Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

The Real Number System (N.RN)

Cluster Headings	Content Standards
A. Use properties of rational and irrational numbers.	**B.N.RN.A.1**. Use rational and irrational numbers in calculations and in real-world context.

Quantities★ (N.Q)

Cluster Headings	Content Standards
A. Reason quantitatively and use units to solve problems.	**B.N.Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **B.N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling. **B.N.Q.A.3** Solve problems involving squares, square roots of numbers, cubes, and cube roots of numbers.

The Complex Number System (N.CN)

Cluster Headings	Content Standards
A. Perform arithmetic operations with complex numbers.	**B.N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **B.N.CN.A.2** Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards
A. Write expressions in equivalent forms to solve problems.	**B.A.SSE.A.1** Use properties of multiplication and division to solve problems containing scientific notation. **B.A.SSE.A.2** Use algebraic structures to solve problems involving proportional reasoning in real-world context.

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards
A. Perform arithmetic operations on polynomials.	**B.A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
B. Understand the relationship between zeros and factors of polynomials.	**B.A.APR.B.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Creating Equations★ (A.CED)

Cluster Headings	Content Standards
A. Create equations that describe numbers or relationships.	**B.A.CED.A.1** Create equations and inequalities in one variable and use them to solve real-world problems. **B.A.CED.A.2** Create equations in two or more variables to represent relationships between quantities. **B.A.CED.A.3** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards
A. Understand solving equations as a process of reasoning and explain the reasoning.	**B.A.REI.A.1** Build functions and write expressions, equations, and inequalities for common algebra settings leading to a solution in context (e.g., rate and distance problems and problems that can be solved using proportions).
B. Solve equations and inequalities in one variable.	**B.A.REI.B.2** Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
C. Solve systems of equations.	**B.A.REI.C.3** Solve and explain the solutions to a system of equations using a variety of representations including combinations of linear and non-linear equations.
D. Represent and solve equations and inequalities graphically.	**B.A.REI.D.4** Use algebra and geometry to solve problems involving midpoints and distances. **B.A.REI.D.5** Solve a linear inequality using multiple methods and interpret the solution as it applies to the context.

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Understand the concept of a function and use function notation.	**B.F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **B.F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Cluster Headings	Content Standards
B. Interpret functions that arise in applications in terms of the context.	**B.F.IF.B.3** Recognize functions as mappings of an independent variable into a dependent variable. ⋆
C. Analyze functions using different representations.	**B.F.IF.C.4** Graph linear, quadratic, absolute value, and piecewise functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated ones. ⋆ **B.F.IF.C.5** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **B.F.IF.C.6** Use the properties of exponents to interpret expressions for exponential functions.

Geometry

Similarity, Right Triangles and Trigonometry (G.SRT)

Cluster Headings	Content Standards
A. Understand similarity in terms of similarity transformations.	**B.G.SRT.A.1** Apply similar triangles to solve problems, such as finding heights and distances.
B. Define trigonometric ratios and solve problems involving right triangles.	**B.G.SRT.B.2** Apply basic trigonometric ratios to solve right triangle problems. **B.G.SRT.B.3** Apply properties of 30° 60° 90°, 45° 45° 90°, similar, and congruent triangles. **B.G.SRT.B.4** Solve problems involving angles of elevation and angles of depression.

Circles (G.C)

Cluster Headings	Content Standards
A. Find arc lengths and areas of sectors of circles.	**B.G.C.A.1** Apply a variety of strategies to determine the area and circumference of circles after identifying necessary information.

Geometric Measurement and Dimension (G.GMD)

Cluster Headings	Content Standards
A. Visualize relationships between two-dimensional and three-dimensional objects.	**B.G.GMD.A.1** Use relationships involving area, perimeter, and volume of geometric figures to compute another measure. **B.G.GMD.A.2** Use several angle properties to find an unknown angle measure. **B.G.GMD.A.3** Apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures in composite figures (i.e., combinations of basic figures).

Modeling with Geometry (G.MG)

Cluster Headings	Content Standards
A. Apply geometric concepts in modeling situations.	**B.G.MG.A.1** Use appropriate technology to find the mathematical model for a set of non-linear data. **B.G.MG.A.2** Solve problems involving surface area and volume in real-world context.

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards
A. Summarize, represent, and interpret data on a single count or measurement variable.	**B.S.ID.A.1** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**B.S.ID.B.2** Interpret and use data from tables, charts, and graphs.

Cluster Headings	Content Standards
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**B.S.ID.B.3** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a**. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
C. Interpret linear models.	**B.S.ID.C.4** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards
A. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**B.S.CP.A.1** Understand and use basic counting techniques in contextual settings. **B.S.CP.A.2** Compute a probability when the event and/or sample space are not given or obvious. **B.S.CP.A.3** Recognize the concepts of conditional and joint probability expressed in real-world contexts. **B.S.CP.A.4** Recognize the concept of independence expressed in real-world contexts.

Precalculus | P

Precalculus is designed to prepare students for college level STEM focused courses. Students extend their knowledge of the complex number system to use complex numbers in polynomial identities and equations. Topics for student mastery include vectors and matrix quantities, sequences and series, parametric equations, and conic sections. Students use previous knowledge to continue progressing in their understanding of trigonometric functions and using regression equations to model quantitative data.

Precalculus includes the following domains and clusters:

- **Number Expressions**
  - Represent, interpret, compare, and simplify number expressions.
- **The Complex Number System**
  - Perform complex number arithmetic and understand the representation on the complex plane.
  - Use complex numbers in polynomial identities and equations.
- **Vectors and Matrix Quantities**
  - Represent and model with vector quantities.
  - Understand the graphic representation of vectors and vector arithmetic.
  - Perform operations on matrices and use matrices in applications.
- **Sequences and Series**
  - Understand and use sequences and series.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations and nonlinear inequalities.
- **Parametric Equations**
  - Describe and use parametric equations.
- **Conic Sections**
  - Understand the properties of conic sections and apply them to model real-world phenomena.
- **Building Functions**
  - Build new functions from existing functions.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle.
- **Graphing Trigonometric Functions**
  - Model periodic phenomena with trigonometric functions.
- **Applied Trigonometry**
  - Use trigonometry to solve problems.
- **Trigonometric Identities**
  - Apply trigonometric identities to rewrite expressions and solve equations.
- **Polar Coordinates**
  - Use polar coordinates.
- **Model with Data**
  - Model data using regression equations.

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Number and Quantity

Number Expressions (N.NE)

Cluster Headings	Content Standards
A. Represent, interpret, compare, and simplify number expressions.	**P.N.NE.A.1** Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. **P.N.NE.A.2** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★ **P.N.NE.A.3** Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e. **P.N.NE.A.4** Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. **P.N.NE.A.5** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

The Complex Number System (N.CN)

Cluster Headings	Content Standards
A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.1** Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$. **P.N.CN.A.2** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **P.N.CN.A.3** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **P.N.CN.A.4** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + 3i)^3 = 8$ *because* $(-1 + 3i)$ *has modulus 2 and argument 120°.*

A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.5** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
B. Use complex numbers in polynomial identities and equations.	**P.N.CN.B.6** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$. **P.N.CN.B.7** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N.VM)

Cluster Headings	Content Standards
A. Represent and model with vector quantities.	**P.N.VM.A.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes *(e.g.,* $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v*)*. **P.N.VM.A.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **P.N.VM.A.3** Solve problems involving velocity and other quantities that can be represented by vectors.
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.4** Add and subtract vectors. **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **c.** Understand vector subtraction $\boldsymbol{v} – \boldsymbol{w}$ as $\boldsymbol{v} + (–\boldsymbol{w})$, where $–\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	P.N.VM.B.5 Multiply a vector by a scalar. **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **b.** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Cluster Headings	Content Standards
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.6** Calculate and interpret the dot product of two vectors.
C. Perform operations on matrices and use matrices in applications.	**P.N.VM.C.7** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **P.N.VM.C.8** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **P.N.VM.C.9** Add, subtract, and multiply matrices of appropriate dimensions. **P.N.VM.C.10** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **P.N.VM.C.11** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **P.N.VM.C.12** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **P.N.VM.C.13** Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Sequences and Series (A.S)

A. Understand and use sequences and series.	**P.A.S.A.1** Demonstrate an understanding of sequences by representing them recursively and explicitly.
	P.A.S.A.2 Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.
	P.A.S.A.3 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist. a. Determine whether a given arithmetic or geometric series converges or diverges. b. Find the sum of a given geometric series (both infinite and finite). c. Find the sum of a finite arithmetic series.

A. Understand and use sequences and series.	**P.A.S.A.4** Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.
	P.A.S.A.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Reasoning with Equations and Inequalities (A.REI)

A. Solve systems of equations and nonlinear inequalities.	**P.A.REI.A.1** Represent a system of linear equations as a single matrix equation in a vector variable. **P.A.REI.A.2** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **P.A.REI.A.3** Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology. **P.A.REI.A.4** Solve systems of nonlinear inequalities by graphing.

Parametric Equations (A.PE)

A. Describe and use parametric equations. ★	**P.A.PE.A.1** Graph curves parametrically (by hand and with appropriate technology). **P.A.PE.A.2** Eliminate parameters by rewriting parametric equations as a single equation.

Conic Sections (A.C)

A. Understand the properties of conic sections and model real-world phenomena.	**P.A.C.A.1** Display all of the conic sections as portions of a cone. **P.A.C.A.2** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **P.A.C.A.3** From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics. **P.A.C.A.4** Transform equations of conic sections to convert between general and standard form.

Functions

Building Functions (F.BF)

Cluster Headings	Content Standards
A. Build new functions from existing functions.	**P.F.BF.A.1** Understand how the algebraic properties of an equation transform the geometric properties of its graph. *For example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, reflections, and changes in periodicity and amplitude).*
	P.F.BF.A.2 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.
	P.F.BF.A.3 Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*
	P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.
	P.F.BF.A.5 Find inverse functions (including exponential, logarithmic, and trigonometric). **a.** Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $1 / f(x)$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type. **b.** Verify by composition that one function is the inverse of another. **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse. **d.** Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.
	P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Analyze functions using different representations.	**P.F.IF.A.1** Determine whether a function is even, odd, or neither. **P.F.IF.A.2** Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★ **P.F.IF.A.4** Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational). **P.F.IF.A.5** Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$. **P.F.IF.A.6** Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur. **P.F.IF.A.7** Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior. **P.F.IF.A.8** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) for n ≥ 1.*

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards
A. Extend the domain of trigonometric functions using the unit circle.	**P.F.TF.A.1** Convert from radians to degrees and from degrees to radians. **P.F.TF.A.2** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number. **P.F.TF.A.3** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **P.F.TF.A.4** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Graphing Trigonometric Functions (F.GT)

Cluster Headings	Content Standards
A. Model periodic phenomena with trigonometric functions.★	**P.F.GT.A.1** Interpret transformations of trigonometric functions. **P.F.GT.A.2** Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. **P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes. **P.F.GT.A.4** Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions. **P.F.GT.A.5** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **P.F.GT.A.6** Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions. **P.F.GT.A.7** Graph the inverse trigonometric functions and identify their key characteristics. **P.F.GT.A.8** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Geometry

Applied Trigonometry (G.AT)

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.1** Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. **P.G.AT.A.2** Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **P.G.AT.A.3** Derive and apply the formulas for the area of sector of a circle. **P.G.AT.A.4** Calculate the arc length of a circle subtended by a central angle.

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.5** Prove the Laws of Sines and Cosines and use them to solve problems. **P.G.AT.A.6** Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Trigonometric Identities (G.TI)

Cluster Headings	Content Standards
A. Apply trigonometric identities to rewrite expressions and solve equations.★	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **P.G.TI.A.2** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Polar Coordinates (G.PC)

Cluster Headings	Content Standards
A. Use polar coordinates.	**P.G.PC.A.1** Graph functions in polar coordinates. **P.G.PC.A.2** Convert between rectangular and polar coordinates. **P.G.PC.A.3** Represent situations and solve problems involving polar coordinates.★

Statistics and Probability

Model with Data★ (S.MD)

Cluster Headings	Content Standards
A. Model data using regressions equations.	**P.S.MD.A.1** Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real-world phenomena and to make predictions. **P.S.MD.A.2** Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data.

A. Model data using regressions equations.	**P.S.MD.A.3** Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.

Statistics | S

Statistics is designed to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. The major themes in Statistics include: interpreting categorical and quantitative data, conditional probability and other rules of probability, using probability to make decisions, and making inferences and justifying conclusions.

Statistics includes the following domains and clusters:

- **Interpreting Categorical and Quantitative Data**
  - Understand, represent, and use univariate data.
  - Understand, represent, and use bivariate data.
- **Conditional Probability and the Rules of Probability**
  - Understand and apply basic concepts of probability.
  - Use the rules of probability to compare probabilities of compound events in a uniform probability model.
- **Using Probability to Make Decisions**
  - Understand and use discrete probability distributions.
  - Understand the normal probability distribution.
- **Making Inferences and Justifying Conclusions**
  - Know the characteristics of well-defined studies.
  - Design and conduct a statistical experiment to study a problem, then interpret and communicate the outcomes.
  - Make inferences about population parameters based on a random sample from that population.
  - Understand and use confidence intervals.
  - Use distributions to make inferences about a data set.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

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1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Exploring Data

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards
A. Understand, represent, and use univariate data.	**S.ID.A.1** Understand the term 'variable' and differentiate between the data types: measurement, categorical, univariate, and bivariate. **S.ID.A.2** Understand histograms, parallel box plots, and scatterplots, and use them to display and compare data. **S.ID.A.3** Summarize distributions of univariate data. **S.ID.A.4** Compute basic statistics and understand the distinction between a statistic and a parameter. **S.ID.A.5** For univariate measurement data, be able to display the distribution and describe its shape; select and calculate summary statistics. **S.ID.A.6** Recognize how linear transformations of univariate data affect shape, center, and spread. **S.ID.A.7** Analyze the effect of changing units on summary measures. **S.ID.A.8** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **S.ID.A.9** Describe individual performances in terms of percentiles, z-scores, and t-scores.
B. Understand, represent, and use bivariate data.	**S.ID.B.10** Represent and analyze categorical data. **S.ID.B.11** Display and discuss bivariate data where at least one variable is categorical. **S.ID.B.12** For bivariate measurement data, be able to display a scatterplot and describe its shape; use technological tools to determine regression equations and correlation coefficients. **S.ID.B.13** Identify trends in bivariate data; find functions that model the data and that transform the data so that they can be modeled.

Probability

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards
A. Understand and apply basic concepts of probability.	**S.CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S.CP.A.2** Use permutations and combinations to compute probabilities of compound events and solve problems. **S.CP.A.3** Demonstrate an understanding of the Law of Large Numbers (Strong and Weak).
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**S.CP.B.4** Demonstrate an understanding of the addition rule, the multiplication rule, conditional probability, and independence. **S.CP.B.5** Apply the general Multiplication Rule in a uniform probability model, *P(A and B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.

Probability Distributions

Using Probability to Make Decisions (S.MD)

Cluster Headings	Content Standards
A. Understand and use discrete probability distributions.	**S.MD.A.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **S.MD.A.2** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **S.MD.A.3** Design a simulation of random behavior and probability distributions (e.g., drawing by lots, using a random number generator and using the results to make fair decisions.) **S.MD.A.4** Analyze discrete random variables and their probability distributions, including binomial and geometric.

Cluster Headings	Content Standards
A. Understand and use discrete probability distributions.	**S.MD.A.5** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
	S.MD.A.6 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
	S.MD.A.7 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
	S.MD.A.8 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
B. Understand the normal probability distribution.	**S.MD.B.9** Calculate the mean (expected value) and standard deviation of both a random variable and a linear transformation of a random variable. **S.MD.B.10** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Sampling and Experimentation

Making Inferences and Justifying Conclusions (S.IC)

Cluster Headings	Content Standards
A. Know the characteristics of well-designed studies.	**S.IC.A.1** Understand the differences among various kinds of studies and which types of inferences can be legitimately drawn from each. **S.IC.A.2** Compare census, sample survey, experiment, and observational study. **S.IC.A.3** Describe the role of randomization in surveys and experiments. **S.IC.A.4** Describe the role of experimental control and its effect on confounding. **S.IC.A.5** Identify bias in sampling and determine ways to reduce it to improve results. **S.IC.A.6** Describe the sampling distribution of a statistic and define the standard error of a statistic. **S.IC.A.7** Demonstrate an understanding of the Central Limit Theorem.
B. Design and conduct a statistical experiment to study a problem, then interpret and communicate the outcomes.	**S.IC.B.8** Select a method to collect data and plan and conduct surveys and experiments. **S.IC.B.9** Compare and use sampling methods, including simple random sampling, stratified random sampling, and cluster sampling. **S.IC.B.10** Test hypotheses using appropriate statistics. **S.IC.B.11** Analyze results and make conclusions from observational studies, experiments, and surveys. **S.IC.B.12** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
C. Make inferences about population parameters based on a random sample from that population.	**S.IC.C.13** Develop and evaluate inferences and predictions that are based on data. **S.IC.C.14** Use properties of point estimators, including biased/unbiased, and variability.
D. Understand and use confidence intervals.	**S.IC.D.15** Understand the meaning of confidence level, of confidence intervals, and the properties of confidence intervals.

Cluster Headings	Content Standards
D. Understand and use confidence intervals.	**S.IC.D.16** Construct and interpret a large sample confidence interval for a proportion and for a difference between two proportions. **S.IC.D.17** Construct the confidence interval for a mean and for a difference between two means.
E. Use distributions to make inferences about a data set.	**S.IC.E.18** Apply the properties of a Chi-square distribution in appropriate situations in order to make inferences about a data set. **S.IC.E.19** Apply the properties of the normal distribution in appropriate situations in order to make inferences about a data set. **S.IC.E.20** Interpret the t-distribution and determine the appropriate degrees of freedom.

Applied Mathematical Concepts | AM

Applications and modeling using mathematics are the primary foci of this course.

Sample potential applications for topics are listed below:

Counting, Combinatorics, and Probability

- Counting hands of cards, code words, license plates, phone numbers, make-up of committees, etc.
- Probabilities associated with games (such as using games from *The Price is Right*)

Financial Math

- Amortization and loans (copayments, credit cards, loans, etc.)
- Compound interest; comparing payments, interest rates, length of loan period, investments, etc.
- Computing taxes
- Computing paychecks (deductions, social security payments, etc.)
- Comparing insurance plans (term vs. whole life)
- Annuities

Linear Programming

- Maximizing capacity while minimizing costs

Applied Mathematical Concepts includes the following domains and clusters:

- **Financial Mathematics**
  - Use financial mathematics to solve problems.
  - Use financial mathematics to make decisions.
  - Determine appropriate models to solve contextual problems.
- **Linear Programming**
  - Use linear programming techniques to solve real-world problems.
  - Solve real-world optimization problems.
- **Logic and Boolean Algebra**
  - Use logic and Boolean Algebra in real-world situations.
  - Apply Boolean Algebra to real-world network problems.
- **Problem Solving**
  - Apply problem solving techniques to real-world problems.
- **Investigate Logic**
  - Use logic to make arguments and solve problems.
  - Determine the validity of arguments.
- **Organize and Interpret Data**
  - Analyze data from multiple viewpoints and perspectives.
- **Counting and Combinatorial Reasoning**
  - Apply probability and counting principles to real-world situations.
  - Use combinatorial reasoning to solve real-world problems.
- **Normal Probability Distribution**
  - Work with the normal distribution in real-world situations.
- **Understand and Use Confidence Intervals**
  - Work with confidence intervals in real-world situations.

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Number and Quantity

Financial Mathematics (N.NQ)

Cluster Headings	Content Standards
A. Use financial mathematics to solve problems.	**AM.N.NQ.A.1** Define interest, compound interest, annuities, sinking funds, amortizations, annuities, future value, and present value. **AM.N.NQ.A.2** Recognize the importance of applying a financial model to business. **AM.N.NQ.A.3** Determine future value and present value of an annuity. **AM.N.NQ.A.4** Determine the amortization schedule for an annuity and a home mortgage.
B. Use financial mathematics to make decisions.	**AM.N.NQ.B.5** Apply financial mathematics to depreciation schedules. **AM.N.NQ.B.6** Solve contextual problems involving financial decision-making. **AM.N.NQ.B.7** Apply arithmetic and geometric sequences to simple and compound interest, annuities, loans, and amortization. **AM.N.NQ.B.8** Solve problems in mathematics of finance involving compound interest using exponential and logarithmic techniques.
C. Determine appropriate models to solve contextual problems.	**AM.N.NQ.C.9** Know when to use transcendental functions to accomplish various application purposes such as predicting population growth. **AM.N.NQ.C.10** Use orders of magnitude estimates for determining an appropriate model for a contextual situation.

Algebra

Linear Programming (A.LP)

Cluster Headings	Content Standards
A. Use linear programming techniques to solve real-world problems.	**AM.A.LP.A.1** Use mathematical models involving equations and systems of equations to represent, interpret, and analyze quantitative relationships, change in various contexts, and other real-world phenomena. **AM.A.LP.A.2** Read, interpret, and solve linear programming problems graphically and by computational methods.

Cluster Headings	Content Standards
B. Solve real-world optimization problems.	**AM.A.LP.B.3** Use linear programming to solve optimization problems. **AM.A.LP.B.4** Interpret the meaning of the maximum or minimum value in terms of the objective function.

Logic and Boolean Algebra (A.LB)

Cluster Headings	Content Standards
A. Use logic and Boolean Algebra in real-world situations.	**AM.A.LB.A.1** Develop the symbols and properties of Boolean algebra; connect Boolean algebra to standard logic. **AM.A.LB.A.2** Construct truth tables to determine the validity of an argument.
B. Apply Boolean Algebra to real-world network problems.	**AM.A.LB.B.3** Analyze basic electrical networks; compare the networks to Boolean Algebra configurations. **AM.A.LB.B.4** Develop electrical networks and translate them into Boolean Algebra equations.

Problem Solving (A.PS)

Cluster Headings	Content Standards
A. Apply problem solving techniques to real-world situations.	**AM.A.PS.A.1** Apply problem solving strategies to real-world situations. *Strategies include, but are not limited to: making orderly lists or tables, drawing diagrams, considering simpler problems, looking for patterns, working backwards, guess and check, using logical reasoning, etc.*

Geometry and Measurement

Investigate Logic (G.L)

Cluster Headings	Content Standards
A. Use logic to make arguments and solve problems.	**AM.G.L.A.1** Define the order of operations for the logical operators. **AM.G.L.A.2** Define conjunction, disjunction, negation, conditional, and biconditional. **AM.G.L.A.3** Solve a variety of logic puzzles.

Cluster Headings	Content Standards
A. Use logic to make arguments and solve problems.	**AM.G.L.A.4** Construct and use a truth table to draw conclusions about a statement.
B. Determine the validity of arguments.	**AM.G.L.B.5** Apply the laws of logic to judge the validity of arguments. **AM.G.L.B.6** Give counterexamples to disprove statements. **AM.G.L.B.7** Analyze arguments with quantifiers through the use of Venn diagrams. **AM.G.L.B.8** Represent logical statements with networks.

Data Analysis, Statistics, and Probability

Organize and Interpret Data (D.ID)

Cluster Headings	Content Standards
A. Analyze data from multiple viewpoints and perspectives.	**AM.D.ID.A.1** Organize data for problem solving. **AM.D.ID.A.2** Use a variety of counting methods to organize information, determine probabilities, and solve problems. **AM.D.ID.A.3** Translate from one representation of data to another, e.g., a bar graph to a circle graph. **AM.D.ID.A.4** Calculate and interpret statistical problems using measures of central tendency and graphs. **AM.D.ID.A.5** Calculate expected value, e.g., to determine the fair price of an investment. **AM.D.ID.A.6** Analyze survey data using Venn diagrams. **AM.D.ID.A.7** Evaluate and compare two investments or strategies, where one investment or strategy is safer but has lower expected value. Include large and small investments and situations with serious consequences.

Counting and Combinatorial Reasoning (D.CR)

Cluster Headings	Content Standards
A. Apply probability and counting principles to real-world situations.	**AM.D.CR.A.1** Use permutations, combinations, and the multiplication principle to solve counting problems. **AM.D.CR.A.2** Design and interpret simple experiments using tree-diagrams, permutations, and combinations. **AM.D.CR.A.3** Apply counting principles to probabilistic situations involving equally likely outcomes. **AM.D.CR.A.4** Solve counting problems by using Venn diagrams and show relationships modeled by the Venn diagram. **AM.D.CR.A.5** Use permutations and combinations to compute probabilities of compound events and solve problems.
B. Use combinatorial reasoning to solve real-world problems.	**AM.D.CR.B.6** Apply the Law of Large numbers to contextual situations.
	AM.D.CR.B.7 Discuss the various examples and consequences of innumeracy; consider poor estimation, improper experimental design, inappropriate comparisons, and scientific notation comparisons.
	AM.D.CR.B.8 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
	AM.D.CR.B.9 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **AM.D.CR.B.10** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Normal Probability Distribution (D.ND)

Cluster Headings	Content Standards
A. Work with the normal distribution in real-world situations.	**AM.D.ND.A.1** Calculate the mean (expected value) and standard deviation of both a random variable and a linear transformation of a random variable.

Cluster Headings	Content Standards
A. Work with the normal distribution in real-world situations.	**AM.D.ND.A.2** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Understand and Use Confidence Intervals (D.CI)

Cluster Headings	Content Standards
A. Work with confidence intervals in real-world situations.	**AM.D.CI.A.1** Understand the meaning of confidence level, of confidence intervals, and the properties of confidence intervals. **AM.D.CI.A.2** Construct and interpret a large sample confidence interval for a proportion and for a difference between two proportions. **AM.D.CI.A.3** Construct the confidence interval for a mean and for a difference between two means.

Calculus | C

Calculus is designed for students interested in STEM-based careers and builds on the concepts studied in precalculus. The study of calculus on the high school level includes a study of limits, derivatives, and an introduction to integrals.

Calculus includes the following domains and clusters:

- **Limits of Functions**
  - Understand the concept of the limit of a function.
- **Behavior of Functions**
  - Describe the asymptotic and unbounded behavior of functions.
- **Continuity**
  - Develop an understanding of continuity as a property of functions.
- **Understand the Concept of the Derivative**
  - Demonstrate an understanding of the derivative.
  - Understand the derivative at a point.
- **Computing and Applying Derivatives**
  - Apply differentiation techniques.
  - Use first and second derivatives to analyze a function.
  - Apply derivatives to solve problems.
- **Understanding Integrals**
  - Demonstrate understanding of a definite integral.
  - Understand and apply the Fundamental Theorem of Calculus.
- **Calculate and Apply Integrals**
  - Apply techniques of antidifferentiation.
  - Apply integrals to solve problems.

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Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Functions, Graphs, and Limits

Limits of Functions (F.LF)

Cluster Headings	Content Standards
A. Understand the concept of the limit of a function.	**C.F.LF.A.1** Calculate limits (including limits at infinity) using algebra. **C.F.LF.A.2** Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piece-wise functions. **C.F.LF.A.3** Draw a sketch that illustrates the definition of the limit; develop multiple real-world scenarios that illustrate the definition of the limit.

Behavior of Functions (F.BF)

Cluster Headings	Content Standards
A. Describe the asymptotic and unbounded behavior of functions.	**C.F.BF.A.1** Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **C.F.BF.A.2** Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior.

Continuity (F.C)

Cluster Headings	Content Standards
A. Develop an understanding of continuity as a property of functions	**C.F.C.A.1** Define continuity at a point using limits; define continuous functions. **C.F.C.A.2** Determine whether a given function is continuous at a specific point. **C.F.C.A.3** Determine and define different types of discontinuity (point, jump, infinite) in terms of limits. **C.F.C.A.4** Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions.

Derivatives

Understand the Concept of the Derivative (D.CD)

Cluster Headings	Content Standards
A. Demonstrate an understanding of the derivative.	**C.D.CD.A.1** Represent and interpret the derivative of a function graphically, numerically, and analytically. **C.D.CD.A.2** Interpret the derivative as an instantaneous rate of change. **C.D.CD.A.3** Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph. **C.D.CD.A.4** Demonstrate the relationship between differentiability and continuity.
B. Understand the derivative at a point.	**C.D.CD.B.5** Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear). **C.D.CD.B.6** Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values. **C.D.CD.B.7** Write the equation of the line tangent to a curve at a given point. **C.D.CD.B.8** Apply the Mean Value Theorem. **C.D.CD.B.9** Understand Rolle's Theorem as a special case of the Mean Value Theorem.

Computing and Applying Derivatives (D.AD)

Cluster Headings	Content Standards
A. Apply differentiation techniques.	**C.D.AD.A.1** Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules. **C.D.AD.A.2** Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric). **C.D.AD.A.3** Calculate the derivatives of sums, products, and quotients of basic functions. **C.D.AD.A.4** Apply the chain rule to find the derivative of a composite function.

Cluster Headings	Content Standards
A. Apply differentiation techniques.	**C.D.AD.A.5** Implicitly differentiate an equation in two or more variables. **C.D.AD.A.6** Use implicit differentiation to find the derivative of the inverse of a function.
B. Use first and second derivatives to analyze a function.	**C.D.AD.B.7** Relate the increasing and decreasing behavior of *f* to the sign of *f'* both analytically and graphically. **C.D.AD.B.8** Use the first derivative to find extrema (local and global). **C.D.AD.B.9** Analytically locate the intervals on which a function is increasing, decreasing, or neither. **C.D.AD.B.10** Relate the concavity of *f* to the sign of *f"* both analytically and graphically. **C.D.AD.B.11** Use the second derivative to find points of inflection as points where concavity changes. **C.D.AD.B.12** Analytically locate intervals on which a function is concave up, concave down, or neither. **C.D.AD.B.13** Relate corresponding characteristics of the graphs of *f*, *f'*, and *f"*. **C.D.AD.B.14** Translate verbal descriptions into equations involving derivatives and vice versa.
C. Apply derivatives to solve problems.	**C.D.AD.C.15** Model rates of change, including related rates problems. In each case, include a discussion of units. **C.D.AD.C.16** Solve optimization problems to find a desired maximum or minimum value. **C.D.AD.C.17** Use differentiation to solve problems involving velocity, speed, and acceleration. **C.D.AD.C.18** Use tangent lines to approximate function values and changes in function values when inputs change (linearization).

Integrals

Understanding Integrals (I.UI)

Cluster Headings	Content Standards
A. Demonstrate understanding of a definite integral.	**C.I.UI.A.1** Define the definite integral as the limit of Riemann sums and as the net accumulation of change. **C.I.UI.A.2** Correctly write a Riemann sum that represents the definition of a definite integral. **C.I.UI.A.3** Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values.
B. Understand and apply the Fundamental Theorem of Calculus.	**C.I.UI.B.4** Recognize differentiation and antidifferentiation as inverse operations. **C.I.UI.B.5** Evaluate definite integrals using the Fundamental Theorem of Calculus. **C.I.UI.B.6** Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable. **C.I.UI.B.7** Apply basic properties of definite integrals (e.g. additive, constant multiple, translations).

Calculate and Apply Integrals (I.AI)

Cluster Headings	Content Standards
A. Apply techniques of antidifferentiation.	**C.I.AI.A.1** Develop facility with finding antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric). **C.I.AI.A.2** Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals). **C.I.AI.A.3** Find specific antiderivatives using initial conditions.

Cluster Headings	Content Standards
B. Apply integrals to solve problems.	**C.I.AI.B.4** Use a definite integral to find the area of a region. **C.I.AI.B.5** Use a definite integral to find the volume of a solid formed by rotating a region around a given axis. **C.I.AI.B.6** Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay).