SUMMER MATH PACKET FOR STUDENTS RISING TO AP CALCULUS

Provided by Cox Math Tutoring

Directions: Complete the following problems without the use of a calculator.

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1. **Simplify, using only positive exponents.**

1.1. $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]^{-3}$

1.2. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

1.3. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

1.4. $\left(\frac{1}{x^{-2}}+\frac{1}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

2. Solve the following absolute value inequalities.

2.1. $|x-3| \leq 4$

2.2. $|3x-4| > -2$

3. Solve the following quadratic inequalities.

3.1. $x^2 - 3x \geq 10$

3.2. $x^3 + 4x^2 - x \geq 4$

3.3. $2sin^2 x \geq sinx,\ 0 \leq x < 2\pi$

4. Solve the following rational inequality.

4.1. $\frac{2x-1}{3x-2} \leq 1$

5. Factor completely.

5.1. $27x^3 - 125y^3$

5.2. $x^4 + 11x^2 - 80$

5.3. $2x^2 + 50y^2 - 20xy$

5.4. $x^2 + 12x + 36 - 9y^2$

5.5. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

5.6. $(3x+4)^{-3} * 2x - 5)^3 + (3x+4)^{-2}(2x-5)^2$

5.7. $\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2}$

6. Solve each equation by factoring, completing the square, or the quadratic formula.

6.1. $x^2 + 6x + 4 = 0$

6.2. $2x^2 - (x+2)(x-3) = 12$

6.3. $x - 10\sqrt{x} + 9 = 0$

6.4. $\frac{1}{x^2} - \frac{1}{x} = 6$

6.5. $x^3 + 2x^2 - 3x - 6 = 0$

7. For each function, find the equation of any/all vertical and horizontal asymptotes.

7.1. $y = \frac{x+4}{x^2+1}$

7.2. $y = \frac{x^2-9}{x^3+3x^2-18x}$

7.3. $y = \frac{2x^3}{x^3-1}$

8. Simplify.

8.1. $\frac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

8.2. $\frac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

8.3. $\frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

8.4. $\frac{x^{-3}-x}{x^{-2}-1}$

8.5. $\frac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x=10}+3}$

9. If $f(x) = x^2,\ g(x) = 2x - 1,\ and\ h(x) = 2^x,\ find\ the\ following$:

9.1. $f(g(2))$

9.2. $h(f(-1))$

9.3. $g(f(h(\frac{1}{2})))$

9.4. $g(f(x))$

9.5. $(g \circ g)(x)$

9.6. $(f \circ h)(x)$

10. Rationalize the denominator (10.1) or the numerator (10.2).

10.1. $\frac{3}{\sqrt[4]{(3x)^3}}$

10.2. $\frac{5\sqrt{2}+\sqrt{5}}{5}$

11. Solve for x.

11.1. $\frac{x-5}{x+1} = \frac{3}{5}$

11.2. $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

11.3. $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

11.4. $\frac{x}{x-2} + \frac{2x}{4-x^2} = \frac{5}{x+2}$

11.5. $\frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$

12. Solve each equation for the indicated variable.

12.1. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1;\ for\ a$

12.2. $A = 2\pi r^2 + 2\pi rh;\ \ for\ r > 0$

12.3. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0;\ for\ x$

13. If $cos\theta = -\frac{5}{13}$, θ in quadrant II, find $sin\theta$ and $tan\theta$.

14. If $cot\theta = 3$, θ in quadrant III, find $sin\theta$ and $cos\theta$.

15. A kite is 100 meters above the ground. If there are 200 meters of string out, find the angle of elevation of the string and the horizontal.

16. Solve each equation on the interval $[0,\ 2\pi)$.

16.1. $cos^2(x) = cos(x)$

16.2. $4sin^2x = 1$

16.3. $2sin^2x + sinx = 1$

16.4. $2sinxcosx + sinx = 0$

16.5. $8cos^2x - 2cosx = 1$

16.6. $sin^2x = cos^2x = 0$

17. Simplify

17.1. $log_2 5 + log_2(x^2 - 1) - log_2(x - 1)$

17.2. $2log_4 9 - log_2 3$

17.3. $3^{2log_3 5}$

18. Determine the equation of each line.

18.1. The line passing through the points $(-1, 3)$ and $(2, -4)$

18.2. The line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$.

18.3. The line through $(2, 3)$ and the midpoint of the segment from $(-1, 4)$ to $(3, 2)$.

19. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find

19.1. The center and the radius

19.2. The equation of the line tangent to the circle at the point $(-2, 5)$

20. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is three times its distance from the point $B(2, -1)$. Determine the equation of the curve.

21. Find the difference quotient, $\frac{f(x+h)-f(x)}{h}$, for:

21.1. $f(x) = 2x + 3$

21.2. $f(x) = \frac{1}{x+1}$

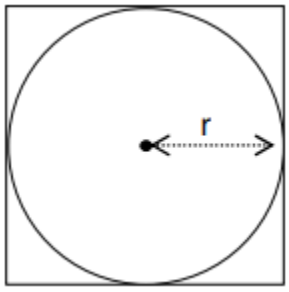
21.3. $f(x) = x^2 + 2x + 1$

22. Find the inverse of each function.

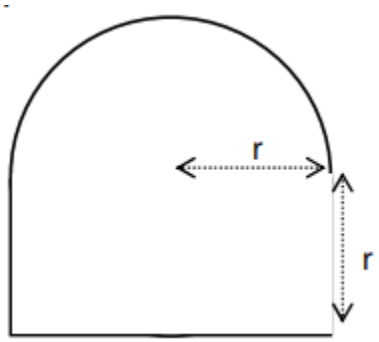
22.1. $f(x) = 2x + 3$

22.2. $f(x) = \frac{x+2}{5x-1}$

23. Find the ratio of the area inside the square but outside the circle to the area of the square.



24. Find a formula for the perimeter of a window with the following shape.



25. A water tank has the shape of an inverted cone (like an ice cream cone). The tank is 10 meters high and has a radius of 3 meters at the top. If the water in the tank is 5 meters deep, what is the surface area of the water in the tank?

26. Let $u = \ < 5, -4 >$ and $v = \ < 1, \ -2 >$. Find $2u - v$.

27. Find a unit vector in the direction of $v =< 3, -1 >$ and write your answer in component form.

28. Given that $P =< 4, -1 >$ and $Q =< 7, -2 >$, find the component form and the magnitude of the vector PQ.

29. Determine whether the vectors U and V are parallel, orthogonal, or neither.
$U =< 5, 3 >, \ V =< \frac{-10}{4}, \frac{-3}{2} >$

30. Evaluate the following limits

30.1. $\lim_{x \to -2} \frac{x^3+8}{x^2-4}$

30.2. $\lim_{x \to 0} \frac{x}{x^2}$

30.3. $\lim_{h \to 0} \frac{1}{h}\left(\frac{1}{2+h} - \frac{1}{2}\right)$